

(1) In  $(\xi, \eta)$  coordinate system:

$$\begin{pmatrix} \delta_\xi \\ \delta_\eta \end{pmatrix} = - \begin{pmatrix} k_\xi & 0 \\ 0 & k_\eta \end{pmatrix} \begin{pmatrix} \frac{\partial T}{\partial \xi} \\ \frac{\partial T}{\partial \eta} \end{pmatrix} \quad \text{i.e.} \quad \underbrace{\begin{cases} \delta_\xi = -k_\xi \frac{\partial T}{\partial \xi} \\ \delta_\eta = -k_\eta \frac{\partial T}{\partial \eta} \end{cases}}$$

The directional cosine between  $(\xi, \eta)$  and  $(x, y)$ :

	$ox$	$oy$
$\partial \xi$	$\cos \beta$	$\sin \beta$
$\partial \eta$	$-\sin \beta$	$\cos \beta$

Then we have:

$$\begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \delta_\xi \\ \delta_\eta \end{pmatrix} \quad \text{i.e.} \quad \underbrace{\begin{cases} \delta_x = \cos \beta \delta_\xi - \sin \beta \delta_\eta \\ \delta_y = \sin \beta \delta_\xi + \cos \beta \delta_\eta \end{cases}}_{(\text{Can be obtained from projection})}$$

$$\text{So: } \begin{cases} \delta_x = \cos \beta \left( -k_\xi \frac{\partial T}{\partial \xi} \right) - \sin \beta \left( -k_\eta \frac{\partial T}{\partial \eta} \right) \\ \delta_y = \sin \beta \left( -k_\xi \frac{\partial T}{\partial \xi} \right) + \cos \beta \left( -k_\eta \frac{\partial T}{\partial \eta} \right) \end{cases}$$

$$\text{i.e. } \underbrace{\begin{cases} \delta_x = -k_\xi \cos \beta \frac{\partial T}{\partial \xi} + k_\eta \sin \beta \frac{\partial T}{\partial \eta} \\ \delta_y = -k_\xi \sin \beta \frac{\partial T}{\partial \xi} - k_\eta \cos \beta \frac{\partial T}{\partial \eta} \end{cases}}$$

Note:  $\begin{cases} \frac{\partial T}{\partial \xi} = \cos \beta \frac{\partial T}{\partial x} + \sin \beta \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial \eta} = -\sin \beta \frac{\partial T}{\partial x} + \cos \beta \frac{\partial T}{\partial y} \end{cases}$

$$\text{So: } \underbrace{\begin{cases} \delta_x = - (k_\xi \cos^2 \beta + k_\eta \sin^2 \beta) \frac{\partial T}{\partial x} - (k_\xi - k_\eta) \sin \beta \cos \beta \frac{\partial T}{\partial y} \\ \delta_y = - (k_\xi - k_\eta) \sin \beta \cos \beta \frac{\partial T}{\partial x} - (k_\xi \sin^2 \beta + k_\eta \cos^2 \beta) \frac{\partial T}{\partial y} \end{cases}}$$

(2) The heat conduction equation:  $\rho C \frac{\partial T}{\partial t} = -\nabla \cdot \vec{q} + g$

\* In  $(\xi, \eta)$  system:  $\begin{cases} q_\xi = -k_\xi \frac{\partial T}{\partial \xi} \\ q_\eta = -k_\eta \frac{\partial T}{\partial \eta} \end{cases}$

$$\text{so: } \nabla \cdot \vec{q} = -\frac{\partial}{\partial \xi} (k_\xi \frac{\partial T}{\partial \xi}) - \frac{\partial}{\partial \eta} (k_\eta \frac{\partial T}{\partial \eta})$$

$$\text{i.e.: } \nabla \cdot \vec{q} = -k_\xi \frac{\partial^2 T}{\partial \xi^2} - k_\eta \frac{\partial^2 T}{\partial \eta^2} \quad (k_\xi, k_\eta \text{ are constants.})$$

Therefore:  $\rho C \frac{\partial T}{\partial t} = k_\xi \frac{\partial^2 T}{\partial \xi^2} + k_\eta \frac{\partial^2 T}{\partial \eta^2} + g(\xi, \eta, t)$

\* In  $(x, y)$  system:  $\begin{cases} q_x = -(k_\xi \cos^2 \beta + k_\eta \sin^2 \beta) \frac{\partial T}{\partial x} - (k_\xi - k_\eta) \sin \beta \cos \beta \frac{\partial T}{\partial y} \\ q_y = -(k_\xi - k_\eta) \sin \beta \cos \beta \frac{\partial T}{\partial x} - (k_\xi \sin^2 \beta + k_\eta \cos^2 \beta) \frac{\partial T}{\partial y} \end{cases}$

$$\text{so: } \nabla \cdot \vec{q} = -(k_\xi \cos^2 \beta + k_\eta \sin^2 \beta) \frac{\partial^2 T}{\partial x^2} - (k_\xi - k_\eta) \sin \beta \cos \beta \frac{\partial^2 T}{\partial xy} - (k_\xi - k_\eta) \sin \beta \cos \beta \frac{\partial^2 T}{\partial xay} - (k_\xi \sin^2 \beta + k_\eta \cos^2 \beta) \frac{\partial^2 T}{\partial y^2}$$

Therefore:  $\rho C \frac{\partial T}{\partial t} = (k_\xi \cos^2 \beta + k_\eta \sin^2 \beta) \frac{\partial^2 T}{\partial x^2} + (k_\xi \sin^2 \beta + k_\eta \cos^2 \beta) \frac{\partial^2 T}{\partial y^2} + (k_\xi - k_\eta) \sin \beta \cos \beta \frac{\partial^2 T}{\partial xay} + g$